Prediction of material ductility and sheet metal formability in relation to plastic instabilities

Farid ABED-MERAIM

Laboratoire d'Etude des Microstructures et de Mécanique des Matériaux, UMR CNRS 7239, Arts et Métiers ParisTech, 4 rue Augustin Fresnel, 57078 Metz, France

In the literature dealing with plastic instabilities in general, many instability criteria have been developed, and some of them have been extensively applied to sheet metals to investigate their formability limits. Exhaustively reviewing these criteria is difficult, considering the multitude of variants deriving from some of these approaches. However, a review of the literature reveals that the criteria can be classified into at least four distinct categories depending on their fundamental basis and theoretical or physical background.

For stretched sheet metals, two forms of necking, namely diffuse and localized necking, may occur. It has been shown that diffuse necking occurs prior to localized necking, and it is now well recognized that the maximum allowable straining in sheet metal forming is determined by localized necking. For this reason, forming limit diagrams (FLDs) are commonly determined at localization in most of the current formability approaches.

Early instability criteria were based on the maximum force principle (Considère, 1885), and its two-dimensional extension (Swift, 1952) for application to sheet metals. In their original form, these criteria were intended to allow for the prediction of diffuse necking. Later, these maximum-force-based criteria were extended to the prediction of localized necking, and some enhanced versions were developed to account for some key features (Hora, 1996; Mattiason, 2006). Note also that Hill's zero-extension criterion (Hill, 1952), which predicts localized necking on the left-hand side of the FLD, was developed during the same time as Swift's diffuse necking criterion.

Another approach, which postulates a pre-existing defect in the material sheet, was proposed by Marciniak and Kuczynski (1967). This M–K model can be regarded as a complementary approach to Hill's zero-extension criterion, which is only applicable to the left-hand side of the FLD, as no zero-extension direction exists for positive biaxial stretching. However, because localized necking in biaxial stretching is observed in practice, a pre-existing defect has to be introduced in the M–K model to capture this phenomenon, which may provide some justification for this imperfection theory.

In addition to the aforementioned engineering approaches, another category of plastic instability criteria was developed based on a more fundamental background. Drucker and Hill's theory (Drucker, 1956; Hill, 1958), also referred to as the general bifurcation criterion, represents another class of approaches for necking prediction. This condition of positiveness of the second-order work provides a lower bound for all of the bifurcation-based criteria in this category. In the same class of criteria, Valanis (1989) suggested using a limit-point bifurcation criterion, which is less conservative than the general bifurcation criterion but coincides with it within the framework of associative plasticity and small strains. With regard to localized modes of deformation, Stören and Rice (1975) proposed a bifurcation criterion characterized by the singularity of the acoustic tensor, also known as discontinuous bifurcation. It has been shown that this criterion corresponds to the loss of ellipticity of the partial differential equations governing the associated boundary value problem. In the same manner, some authors (e.g., Bigoni and Hueckel, 1991) have suggested the use of the more conservative condition of strong ellipticity, which has been shown to coincide with Rice's criterion within the framework of associative plasticity and small strains. This condition of loss of strong ellipticity is also a special case of Drucker's general bifurcation criterion, in which the bifurcation mode is restricted to localized (compatible) deformation modes.

From this overview of the various approaches pertaining to strain localization criteria and indicators, an interesting observation can be made. While M–K analysis has been widely used in the literature, few applications of Rice's ellipticity loss theory, mainly restricted to plane-stress assumptions, particular loading paths, and simple behavior models, have been attempted in sheet metal forming for quantifying metals in terms of their formability. In this presentation, various results relating to the prediction of plastic flow localization based on bifurcation theory will be shown for different constitutive modeling approaches. Also, comparisons will be conducted for the different approaches of plastic instability prediction. For some approaches, a classification of the criteria will be established, in terms of their conservative nature of prediction. Moreover, similarities or relationships between some approaches and associated criteria will be emphasized, whenever their underlying formulations make it possible.

REFERENCES

A. Considère. *Ann. Ponts et Chaussées,* 9, 574–775, 1885.
H.W. Swift. *J. Mech. Phys. Sol.,* 1, 1–18, 1952.
P. Hora, L. Tong, J. Reissner. In: *Proc. of Numisheet'96,* eds. R. Wagonner et al., 252–256, 1996.
K. Mattiasson, M. Sigvant, M. Larson. In: *IDDRG'06,* eds. Santos & Barata da Rocha, Porto, 1–9, 2006.
R. Hill. *J. Mech. Phys. Sol.,* 1, 19–30, 1952.
Z. Marciniak, K. Kuczynski. *Int. J. Mech. Sci.,* 9, 609–620, 1967.
D.C. Drucker. *Q. Appl. Math.,* 16, 35–42, 1956.
R. Hill. *J. Mech. Phys. Sol.,* 6, 236–249, 1958.
K.C. Valanis. *Acta Mech.*, 79, 113–141, 1989.
S. Stören, J.R. Rice. *J. Mech. Phys. Sol.,* 23, 421–441, 1975.
D. Bigoni, T. Hueckel. *Int. J. Solids Struct.*, 28, 197–213, 1991.
G. Franz, F. Abed-Meraim, J.P. Lorrain, T. Ben Zineb, X. Lemoine, M. Berveiller. *Int. J. of Plasticity*, 25, 205–238, 2009.
B. Haddag, F. Abed-Meraim, T. Balan. *Int. J. of Plasticity*, 25, 1970–1996, 2009.
G. Franz, F. Abed-Meraim, M. Berveiller. *Int. J. of Plasticity*, 48, 1–33, 2013.
F. Abed-Meraim, T. Balan, G. Altmeyer. *Int. J. Adv. Manuf. Tech.*, 71, 1247–1262, 2014.
L.Z. Mansouri, H. Chalal, F. Abed-Meraim. *Mech. of Mater.,* 76, 64–92, 2014.
F. Abed-Meraim, R.H.J. Peerlings, M.G.D. Geers. *Int. J. Appl. Mech.*, 6 (6), 27 pp., 2014.
M. Ben Bettaieb, F. Abed-Meraim. *Int. J. of Plasticity*, 65, 168–190, 2015.
H. Chalal, F. Abed-Meraim. *Mech. of Mater.*, 91, Part 1, 152–166, 2015.
Y. Bouktir, H. Chalal, M. Haddad, F. Abed-Meraim. *Mech. & Design*, 90, 969–978, 2016.
M. Ben Bettaieb, F. Abed-Meraim. *Int. J. Adv. Manuf. Tech.*, 92, 3461–3480, 2017.
H.K. Akpama, M. Ben Bettaieb, F. Abed-Meraim. *Int. J. of Plasticity*, 91, 205–237, 2017.
M. Ben Bettaieb, F. Abed-Meraim. *Int. J. Mech. Sci.*, 123, 177–197, 2017.
Y. Bouktir, H. Chalal, F. Abed-Meraim. *Int. J. Damage Mech.*, 27, 801–839, 2018.
A. Gupta, M. Ben Bettaieb, F. Abed-Meraim, S.R. Kalidindi. *Int. J. of Plasticity*, 103, 168–187, 2018.